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Muon decays in the Earth's atmosphere, time dilatation and relativity of simultaneity

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Abstract

Observation of the decay of muons produced in the Earth's atmosphere by cosmic ray interactions provides a graphic illustration of the counter-intuitive space-time predictions of special relativity theory. Muons at rest in the atmosphere, decaying simultaneously, are subject to a universal time-dilatation effect when viewed from a moving frame and so are also observed to decay simultaneously in all such frames, whereas the decays of muons with different proper frames show relativity of simultaneity when observed from different inertial frames.

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1 Introduction

The present paper is the seventh in a recent series devoted to space-time physics, written by the present author, and posted on the arXiv preprint server. In Ref. [1] the classical 'Rockets-and-String' [2] and 'Pole-and-Barn' [3] paradoxes of special relativity were re-analysed taking into account the distinction between the real and apparent[1] positions of uniformly moving objects. Different results were obtained from the usual text-book interpretations of these experiments and a new causality-violating paradox was pointed out. This paradox, as well as the related 'backwards running clocks' one of Soni [4], was resolved in Ref. [5] where, in order to avoid these paradoxes as well as manifest breakdown of translational invariance, in some applications of the standard space-time Lorentz transformation, the use of a 'local' Lorentz transformation. i.e. one where the transformed event in the moving frame lies at the coordinate origin in this frame, was advocated. When this is done the closely correlated 'relativity of simultaneity' (RS) and 'length contraction' (LC) effects of conventional special relativity theory do not occur. The connection between these effects is explained in Ref. [1]. Ref. [5] also contains a 'mini review' of all experimental tests of special relativity where it is pointed out that, whereas time dilatation is well-confirmed experimentally, no experimental evidence exists for the RS and LC effects. In the later papers [6] and [7] it is explained how the spurious RS and LC effects result from a misuse of the time symbols in the standard space-time Lorentz transformation. Ref. [7] presents the argument in a pedagogical manner, whereas Ref. [6] contains a brief 'executive summary' of it. In Ref. [8] it is shown that the absence of the conventional RS effect follows directly from the Reciprocity Postulate [9, 10], that holds in both Galilean and special relativity, without invoking the Galilean or Lorentz transformations. As discussed in Ref. [11] mis-use of the standard space-time Lorentz transformation in classical electrodynamics leads to incorrect relativistic derivations, in particular of the classical (pre-relativistic) Heaviside formula [12] for the fields of a uniformly moving charge, which is no longer valid in relativistic classical electrodynamics. In Ref. [13], Einstein's classic train/embankment experiment [14], used in many text books to demonstrate the RS effect before introducing the Lorentz transformation, is reanalysed and a conclusion –the absence of any RS effect– contrary to that of Einstein, is found. However a genuine 'relativity of simultaneity' effect is predicted by special relativity in the case that observers in two trains of suitably chosen speeds. as well as an embankment observer are considered. The main aim of the present paper is to present a conceptually similar, but simpler, demonstration of this RS effect (quite distinct from the text-book one derived by incorrect use of the Lorentz transformation) by considering the decay of muons at rest in different inertial frames.

In the following section the necessary formulae for the analysis of the muon decay thought experiment –essentially the prediction of a universal time dilatation effect– are derived from first principles. Here there is considerable overlap with work presented in [6] and [7]. The analysis of the thought experiment presented in Section 3 shows both the absence of the spurious text-book RS effect (muons which decay simultaneously in a common proper frame, are observed to do so in all inertial frames) as well as a genuine

[1] i.e. as naively predicted by the standard space-time Lorentz transformation

relativistic RS effect when the observation of the decays of muons at rest in different inertial frames is considered.

2 Operational meaning of the space-time Lorentz transformation: Rates and spatial separations of moving clocks

The Lorentz transformation (LT) relates observations (x,y,z,t) of the coordinates of space-time events in one inertial frame S, to observations of the coordinates (x',y',z',t') of the same events in another inertial frame S'. As is conventional, the Cartesian spatial coordinate axes of the two frames are parallel, and the origin of the frame S' moves with constant speed, v, along the x-axis. In any actual experiment, times are recorded by clocks and positions specified by marks on fixed rulers (or their equivalent). Therefore, in order to relate the space-time coordinates appearing in the LT to actual physical measurements they must be identified with clock readings and length interval measurements. This can be done in two distinct ways depending on whether the experimenter observing the clocks and performing the length measurements is at rest in S or in S'. In the former case (only events with spatial coordinates along the x-axis are considered so that $y = y' = z = z' = 0$) the appropriate LT is:

$$x' = \gamma_v[x - v\tau] \tag{2.1}$$

$$t' = \gamma_v[\tau - \frac{\beta_v x}{c}] \tag{2.2}$$

and in the latter case is:

$$x = \gamma_v[x' + c\tau'] \tag{2.3}$$

$$t = \gamma_v[\tau' + \frac{\beta_v x'}{c}] \tag{2.4}$$

In these equations $\beta_v \equiv v/c$, $\gamma_v \equiv 1/\sqrt{1-\beta_v^2}$ and c is the speed of light in vacuum. In (2.1) and (2.2) the transformed events lie on the worldline of a clock, C', at rest in S', which is observed from S. The observed time in S registered by C'(which is in motion in this frame) is t', while τ is the time registered by a clock, C, identical to C', but at rest in S. In contrast, in (3) and (4) the transformed events lie on the worldline of C, which is observed from S'. The time t is that registered by C as observed from S' and τ' is the time registered by C' as observed in its own proper frame. Thus two experiments are possible involving one stationary and one moving clock, depending on whether the experimenter performing the space and time measurements is in the rest frame of one, or the other, of the two clocks. To describe both of these experiments, the four different time symbols, τ, τ', t and t', with different operational meanings, are required.

In order to derive predictions, without introducing any specific spatial coordinate system, it is convenient to introduce invariant interval relations [15, 16] that may be derived directly from (2.1) and (2.2) or (2.3) and (2.4):

$$c^2(\Delta t')^2 - (\Delta x')^2 = c^2(\Delta\tau)^2 - (\Delta x)^2 \tag{2.5}$$

$$c^2(\Delta t)^2 - (\Delta x)^2 = c^2(\Delta\tau')^2 - (\Delta x')^2 \tag{2.6}$$

(2.5) connects different events on the worldline of C', (2.6) different events on the worldline of C. Since C' is at rest in S', $\Delta x' = 0$, while the equation of motion of C' in S is $\Delta x = v\Delta\tau$. Substituting these values in (2.5) and using the definition of γ_v, gives:

$$\Delta\tau = \gamma_v \Delta t' \tag{2.7}$$

Similarly, since C is at rest in S, $\Delta x = 0$, and the equation of motion of C in S' is $\Delta x' = -v\Delta\tau'$. Hence (2.6) yields the relation:

$$\Delta\tau' = \gamma_v \Delta t \tag{2.8}$$

(2.7) and (2.8) are expressions of the relativistic Time Dilatation (TD) effect in the two 'reciprocal' experiments that may be performed using the clocks C and C'. They show that, according, to the LT, 'moving clocks run slow' in a universal manner (no spatial coordinates appear in (2.7) and (2.8)). In fact:

$$\frac{\text{rate of moving clock}}{\text{rate of stationary clock}} = \frac{\Delta t'}{\Delta\tau} = \frac{\Delta t}{\Delta\tau'} = \frac{1}{\gamma_v} \tag{2.9}$$

To discuss measurements of the spatial separations of moving clocks, at least two clocks, (say, $C_{A'}$ and $C_{B'}$, at rest in S') must be considered. It is assumed that they lie along the x'-axis separated by the distance L'. It will be convenient to introduce two further identical clocks C_A and C_B at rest in S at such a separation that when the x-coordinates of C_A and $C_{A'}$ coincide, so do those of C_B and $C_{B'}$. Further suppose that all the clocks are first stopped (i.e. no longer register time) and are set to zero. They are then all restarted at the instant that C_A and $C_{A'}$ as well as C_B and $C_{B'}$ have the same x-coordinates. Such a procedure has been introduced by Mansouri and Sexl [17] and termed 'system external clock sychronisation'. In this case, the time intervals in (2.7) and (2.8) may be replaced by registered clock times: $\Delta\tau \to \tau$ etc. Also the space-like invariant interval relation connecting arbitary points on the wordlines of $C_{A'}$ and $C_{B'}$ may be written as:

$$\begin{aligned}(\Delta s)^2 &\equiv (L')^2 - c^2[t'(C_{B'}) - t'(C_{A'})]^2 \\ &= [x(C_{B'}) - x(C_{A'})]^2 - c^2[\tau(C_B) - \tau(C_A)]^2 \end{aligned} \tag{2.10}$$

The spatial separation of $C_{A'}$ and $C_{B'}$ in the frame S, L, is defined as the value of $x(C_{B'}) - x(C_{A'})$ at some particular instant in S. For this $\tau(C_B) = \tau(C_A)$, so that (2.10) may be written:

$$(\Delta s)^2 \equiv (L')^2 - c^2[t'(C_{B'}) - t'(C_{A'})]^2 = L^2 \tag{2.11}$$

After the 'external synchronisation' procedure (2.7) may be written: $\tau(C_A) = \gamma_v t'(C_{A'})$ and $\tau(C_B) = \gamma_v t'(C_{B'})$ for the clocks $C_{A'}$ and $C_{B'}$ respectively. Then, if $\tau(C_B) = \tau(C_A)$, necessarily $t'(C_{B'}) = t'(C_{A'})$ and (2.11) simplifies to:

$$\Delta s \equiv L' = L \tag{2.12}$$

The measured spatial separation of the clocks is therefore a Lorentz invariant quantity [18] that is the same in all inertial frames. Thus there is no 'relativistic length contraction'

effect. Also, since $\tau(\mathrm{C}_B) = \tau(\mathrm{C}_A)$ requires that also $t'(\mathrm{C}_{B'}) = t'(\mathrm{C}_{A'})$ there is here no 'relativity of simultaneity' effect either. How these correlated, spurious, effects arise from misinterpretation of meaning of the time symbols in the LT is explained elsewhere [5, 6, 7, 8]. A genuine relativity of simultaneity effect arising in observations of muon decays, occuring in different inertial frames, is, however, described in the following section. The clocks A', A and B, discussed in an abstract fashion above, are physically realised by introducing the muons $\mu_{\mathrm{A}'}$, μ_{A} and μ_{B} . Each muon constitutes an independent clock whose proper time, T, is signalled in an arbitary inertial frame by observation of the muon decay event: $\mu \rightarrow e\nu\bar{\nu}$.

3 Muons are clocks that demonstrate time dilatation and relativity of simultaneity

Muon decays constitute an excellent laboratory for testing the predictions of special relativity. For example, the TD effect of Eqn(2.7) was experimentally confirmed at the per mille level of relative precision in the ultrarelativisic domain ($\gamma_v \simeq 30$) by observation of the decay of muons in the storage ring of the last CERN muon $g-2$ experiment [19]. In the present paper, it is shown that thought experiments involving muons provide a graphic illustration of the predicted space-time behaviour, in special relativity, of clocks in different inertial frames.

Unlike most other unstable particles, muons are particularly suitable for precise tests of the TD effect because of the ease of their production from pion decay and long mean lifetime of 2.2 μs. The former yields high events statistics and the latter the possiblity of precise time interval measurements using accurate clocks in the laboratory frame [19].

The thought experiment developed in the present paper is an elaboration of the well-known demonstration that the very presence of cosmic muons at the Earth's surface is, by itself, sufficient to demonstrate the existence of the TD effect [20, 21, 22, 23]. Muons are produced predominantly by the weak decay of charged pions $\pi^{\pm} \rightarrow \mu^{\pm}\nu$. The velocity of the muon, v_{μ}, depends upon that of the parent muon, v_{π}, and the centre-of-mass decay angle, $\theta^{\star}$. If the pion has the same velocity, $v_{\mu}^{\star} = c(m_{\pi}^2 - m_{\mu}^2)/(m_{\pi}^2 + m_{\mu}^2)$, as the muon in the pion rest frame, (corresponding to a pion momentum of 49.5 MeV/c) and $\cos\theta^{\star} = -1$, the muon is produced at rest in the laboratory system. The maximum muon decay energy E_{μ}^{max} correponds to $\cos\theta^{\star} = 1$ and is given, in terms of the parent pion energy E_{π}, and the pion velocity $v_{\pi} = c\beta_{\pi}$, by the relation:

$$E_{\mu}^{max} = E_{\pi}\frac{[m_{\pi}^2(1+\beta_{\pi}) + m_{\mu}^2(1-\beta_{\pi})]}{2m_{\pi}^2} \qquad (3.1)$$

For ultra-relativistic parent pions with $\beta_{\pi} \simeq 1$, $E_{\mu}^{max} \simeq E_{\pi}$.

Due to the thickness of the Earth's atmosphere, the majority of interactions of primary cosmic protons, that produce the parent pions of cosmic muons, occur at high altitude, $\simeq$ 20 km above the Earth's surface. A muon with speed close to that of light then takes at least $\simeq 700\ \mu s$ to reach the surface of the Earth. This may be compared with the muon

mean lifetime of 2.2 μs. Without the TD effect, only a fraction $\exp[-700/2.2] \simeq 10^{-138}$ of the muons produced at altitude would reach the Earth's surface. However a 10 GeV muon, which has $\gamma_v \simeq 94$, has a 3.5 % probability to reach the Earth's surface, before decaying, when the TD effect is taken into account.

In the thought experiment considered here it is assumed that two muons $\mu_{\rm A}$ and $\mu_{\rm A'}$ are produced simultaneously at the same point A (see Fig.1a) by decay of pions from a primary cosmic ray interaction with the nucleus of a gas atom of the atmosphere. The muon $\mu_{\rm A}$ is produced at rest in the atmosphere (inertial frame S) while $\mu_{\rm A'}$ is produced with velocity $v = c\beta_v = \sqrt{3}/2$, so that $\gamma_v = 2$. It happens that both muons decay after time T in their proper frames. Because of the TD effect, the muon $\mu_{\rm A'}$ will then be seen by an observer at rest in the atmosphere to decay after time $\tau = \gamma_v T = 2T$ at a point B at a distance $L = 2Tv = 2.28$km from A. It is also supposed that at the same time, $\tau = 0$, that $\mu_{\rm A}$ and $\mu_{\rm A'}$ are created another muon, $\mu_{\rm B}$, (also with proper decay lifetime T) is created at rest in the atmosphere at the point B, by decay of pion from another primary cosmic ray interaction. Since $\mu_{\rm A}$ and $\mu_{\rm B}$ are at rest in the atmosphere and have no TD effect, they will decay simultaneously at $\tau = T$ (Fig.1b) in the frame S. At this instant the muon $\mu_{\rm A'}$ is still undecayed and is at the point M, midway between A and B, When $\mu_{\rm A'}$ decays (Fig.1c) $\mu_{\rm A}$ and $\mu_{\rm B}$ no longer exist, however the centres of mass of their, by now distant, decay products e, ν and $\bar{\nu}$ still remain at the points A and B.

The sequence of events that would be seen by an observer in the rest frame, S', of $\mu_{\rm A'}$ is shown in Fig.2. In S', $\mu_{\rm A}$ and $\mu_{\rm B}$ move to the left with velocity v. The configuation at $\tau = \tau' = 0$ is shown in Fig.2a. At time $\tau' = T = L/(2v)$ (Fig.2b) $\mu_{\rm A'}$ decays when it has the same x' coordinate as the point M. The muons $\mu_{\rm A}$ and $\mu_{\rm B}$ are still undecayed. At time $\tau' = \gamma_v T = L/v$ (Fig.2c) $\mu_{\rm A}$ and $\mu_{\rm B}$ are observed to decay simultaneously. At this time A'(the centre of mass of the $\mu_{\rm A'}$ decay products) has the same x-coordinate as the point B in the atmosphere. Note that $\mu_{\rm A}$ and $\mu_{\rm B}$ are observed to decay simultaneously in S' as well as in S. In fact their decays will be observed to be simultaneous in *any* inertial frame. In this case, there is no RS effect as predicted by a conventional (but incorrect) text book application of the LT [5, 6, 7, 8].

Finally, in Fig.3, is shown the same sequence of muon decay events as they would be seen by an observer at rest in the frame S" that is moving parallel to the direction of motion of $\mu_{\rm A'}$ with velocity $w = c^2(\gamma_v - 1)/(v\gamma_v)$ relative to the atmosphere. In S", $\mu_{\rm A'}$ moves with speed w in one direction, while $\mu_{\rm A}$ and $\mu_{\rm B}$ move with speed w in the opposite one (see Fig.3a). Since the TD effect is now the same for all three muons they will observed from S" to decay simultaneously at the time $\tau'' = \gamma_w T = 0.613L/v$ as shown in Fig.3b.

The muon decay events thus exhibit a genuine 'relativity of simultaneity' effect. The simultaneous decays of $\mu_{\rm A}$ and $\mu_{\rm B}$ in their proper frame S (fig.1b) are also seen to be simultaneous in the frames S' (Fig.2c) and S" (Fig.3b). However, the decay times τ_D, τ'_D. and τ''_D of $\mu_{\rm A'}$ show relativity of simultaneity relative to those of $\mu_{\rm A}$ and $\mu_{\rm B}$:

$$\begin{array}{lccc}
\text{Frame S (Fig.1)} & \tau_D(\mu_{\rm A'}) & > & \tau_D(\mu_{\rm A}) = \tau_D(\mu_{\rm B}) \\
\text{Frame S}'' \text{ (Fig.3)} & \tau''_D(\mu_{\rm A'}) & = & \tau''_D(\mu_{\rm A}) = \tau''_D(\mu_{\rm B}) \\
\text{Frame S}' \text{ (Fig.2)} & \tau'_D(\mu_{\rm A'}) & < & \tau'_D(\mu_{\rm A}) = \tau'_D(\mu_{\rm B})
\end{array}$$

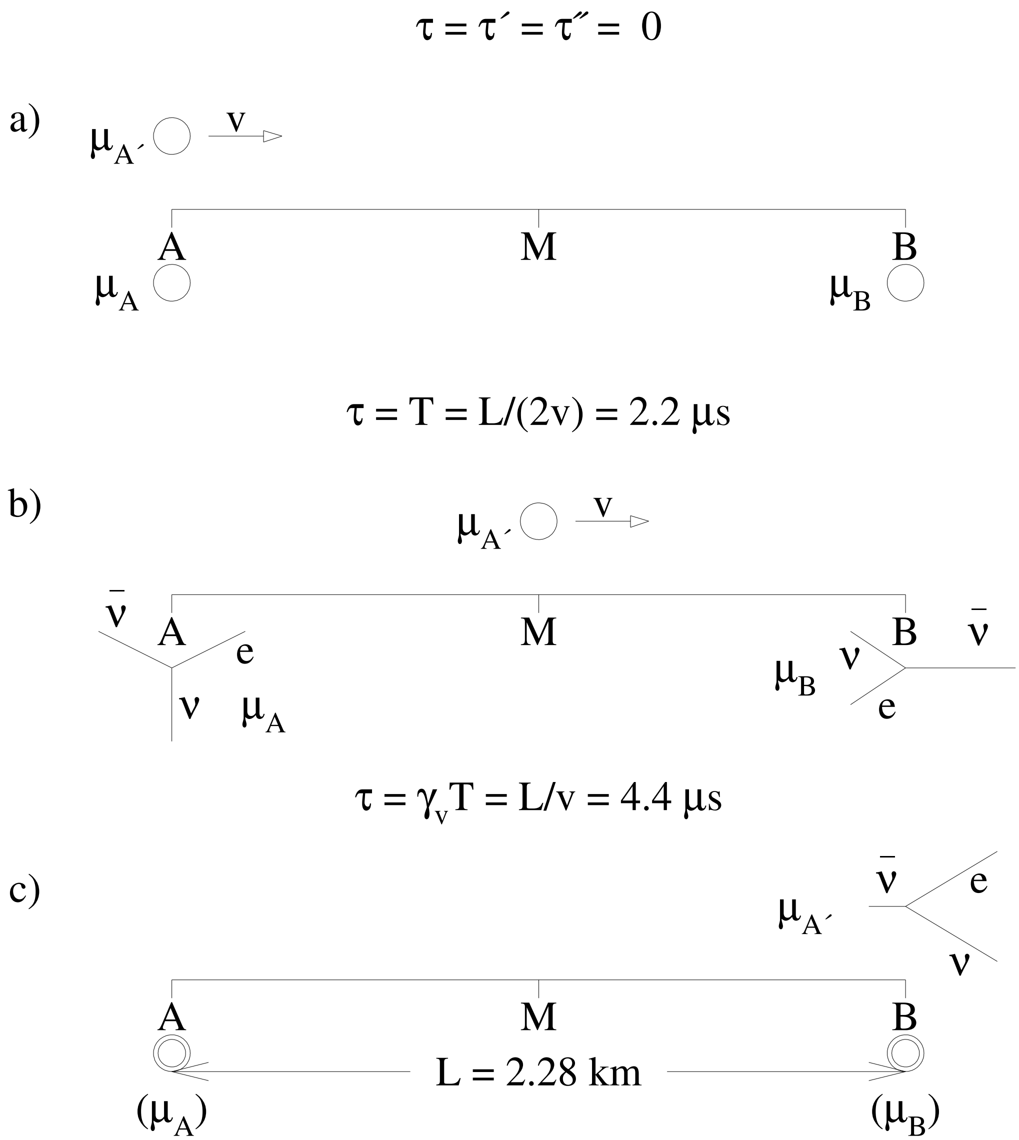

Figure 1: *The sequence of muon decay events as observed from the atmosphere (frame S). a) Muons $\mu_{\mathrm{A}'}$, μ_{A} and μ_{B} are simultaneously created. Muon $\mu_{\mathrm{A}'}$ moves to the right with velocity $v = (\sqrt{3}/2)c$. b) At time $\tau = T$, muons μ_{A} and μ_{B} decay simultaneously. At this time $\mu_{\mathrm{A}'}$ is observed from S to be aligned with the mid-point, M, of A and B. c) At time $\tau = \gamma_v T$, muon $\mu_{\mathrm{A}'}$ is observed to decay. At this time it is at B, the centre of mass of the decay products of μ_{B} . For clarity, the muons are shown displaced vertically.*

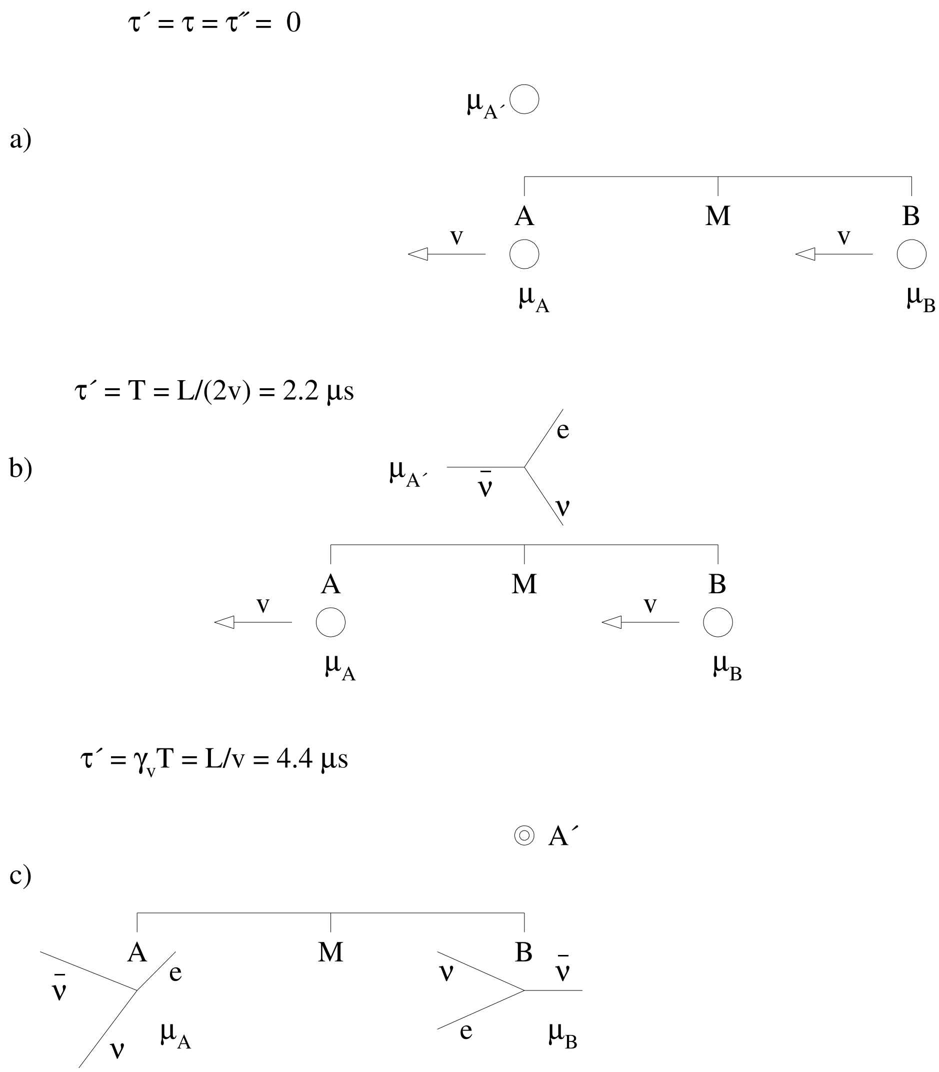

Figure 2: *The sequence of muon decay events as observed in the proper frame (S') of $\mu_{\mathrm{A}'}$. a) Muons $\mu_{\mathrm{A}'}$, μ_{A} and μ_{B} are simultaneously created. Muons μ_{A} and μ_{B} move to the left with velocity $v = (\sqrt{3}/2)c$. b) At time $\tau' = T$, muon $\mu_{\mathrm{A}'}$ decays. At this time it is aligned with the mid-point, M, of A and B. c) At time $\tau' = \gamma_v T$ muon μ_{A} and μ_{B} decay simultaneously. At this time B, is aligned with the point A', the centre of mass of the decay products of $\mu_{\mathrm{A}'}$. For clarity, the muons are shown displaced vertically.*

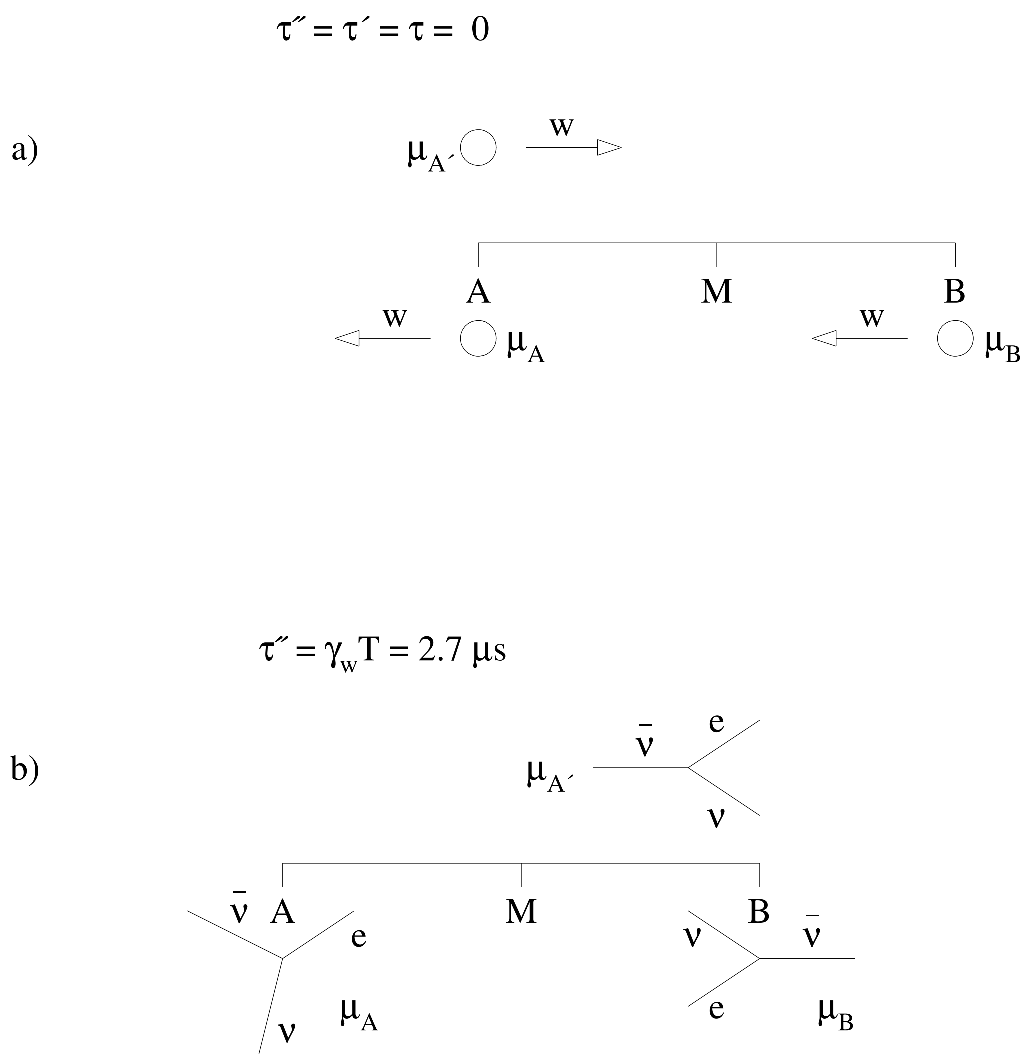

Figure 3: *The sequence of muon decay events as observed from the frame (S") that moves parallel to the direction of motion of $\mu_{A'}$ with the velocity $w = c^2(\gamma_v - 1)/(v\gamma_v) = c/\sqrt{3}$. a) Muons $\mu_{A'}$, μ_A and μ_B are simultaneously created. Muon $\mu_{A'}$ moves to the right with velocity w and μ_A and μ_B move to the left with velocity w. b) At time $\tau'' = \gamma_w T$, all three muons decay simultaneously. In this case, unlike for the reciprocal observations shown in Figs.1c and 2c, $\mu_{A'}$ and μ_B (or the centres of mass of their decay products) are not aligned at the time of the decay of either. For clarity, the muons are shown displaced vertically.*

Frame	$\tilde{\tau}_D(\mu_{\mathrm{A}'})$	$\tilde{x}_D(\mu_{\mathrm{A}'}) - \tilde{x}_{\mathrm{B}}$	$\tilde{\tau}_D(\mu_{\mathrm{B}})$	$\tilde{x}_D(\mu_{\mathrm{B}}) - \tilde{x}_{\mathrm{A}'}$
	Special Relativity			
S	$\gamma_v T$	0	T	$v(\gamma_v - 1)T$
S'	T	$-v(\gamma_v - 1)T$	$\gamma_v T$	0
S"	$\gamma_w T$	$-(v\gamma_v - 2w\gamma_w)T$	$\gamma_w T$	$(v\gamma_v - 2w\gamma_w)T$
	Galilean Relativity			
S,S',S"	T	0	T	0

Table 1: *Decay times and spatial separations of muons $\mu_{\mathrm{A}'}$ and μ_{B} , or the centres of mass of their decay products, in the frames S, S' and S". $\tilde{\tau}$ and $\tilde{x}$ are generic proper times and spatial coordinates. The last row shows the predictions of Galilean relativity, that are the same in all frames.*

A similar pattern of events is seen in a variant of Einstein's train/embankment thought experiment with trains moving at speeds v and w relative to the embankment [13].

Table 1 shows the decay times and relative spatial separations of the muons $\mu_{\mathrm{A}'}$ and μ_{B} (or the centres of mass of their decay products) in the frames S (the atmosphere, or the proper frame of μ_{B}), S' (the $\mu_{\mathrm{A}'}$ proper frame) and S" (a frame moving with velocity w relative to the atmosphere). The symbols $\tilde{\tau}$ and $\tilde{x}$ denote generic proper times or coordinates. e.g. $\tilde{\tau}$ stands for any of τ, τ' and τ''. The differences of proper times of the decay events in different frames or the spatial separation of $\mu_{\mathrm{A}'}$ and μ_{B} at the decay times of either, in these frames, are all of $\mathrm{O}(\beta^2)$, and so vanish in the Galilean limit $c \rightarrow \infty$. The times and separations in this Galilean limit (the same in all frames) are shown in the last row of Table 1. Note that the limit of w as $c \rightarrow \infty$ is $v/2$, so that the spatial separations $x''_D(\mu_{\mathrm{A}'}) - x''_{\mathrm{B}}$ and $x''_D(\mu_{\mathrm{B}}) - x''_{\mathrm{A}'}$ both vanish in this limit.

Inspection of, and reflection upon, Figs.1-3 shows behaviour greatly at variance with intuition derived from Galilean space-time. The same muon decay events appear differently ordered in time and at different spatial separations depending on the frame of observation. These effects are the space-time analogues of the distortions produced by linear perspective in visual perception. Just as the latter are unique for every observer, so the former are unique to every different inertial frame.

Added Note

A revised and re-titled version of the present paper is available [24]. Calculational and conceptual errors in the original paper [25] are explained in this note.

The calculation of the invariance of length intervals based on the invariant interval relation (2.10) is flawed since the times $t'(\mathrm{C}_{A'})$, $t'(\mathrm{C}_{B'})$ do not correspond to those of synchronised clocks in the frame S'. Placing $\mathrm{C}_{A'}$ at the origin of S' and $\mathrm{C}_{B'}$ at $x' = L'$ the correct LT equations for clocks which are synchronised in S' are:

$$x'(\mathrm{C}_{A'}) = \gamma_v[x(\mathrm{C}_{A'}) - v\tau(\mathrm{C}_A)] = 0 \quad (3.2)$$

$$t'(\mathrm{C}_{A'}) = \gamma_v[\tau(\mathrm{C}_A - \frac{\beta_v x(\mathrm{C}_{A'})}{c}] \quad (3.3)$$

$$x'(\mathrm{C}_{B'}) - L' = \gamma_v[x(\mathrm{C}_{B'}) - L - v\tau(\mathrm{C}_B)] = 0 \quad (3.4)$$

$$t'(\mathrm{C}_{B'}) = \gamma_v[\tau(\mathrm{C}_B) - \frac{\beta_v(x(\mathrm{C}_{B'}) - L)}{c}] \quad (3.5)$$

These equations show that $t'(\mathrm{C}_{A'}) = t'(\mathrm{C}_{B'}) = \tau(\mathrm{C}_A) = \tau(\mathrm{C}_B) = 0$ when $x(\mathrm{C}_{A'}) = x(\mathrm{C}_{B'}) - L = 0$, so that all four clocks are synchronised at this instant. The correct invariant interval relation is then not (2.10) but:

$$\begin{aligned}(\Delta s)^2 &\equiv [x'(\mathrm{C}_{B'}) - x'(\mathrm{C}_{A'})]^2 - c^2[t'(\mathrm{C}_{B'}) - t'(\mathrm{C}_{A'})]^2 \\ &= [x(\mathrm{C}_{B'}) - x(\mathrm{C}_{A'})]^2 - c^2[\tau(\mathrm{C}_B) - \tau(\mathrm{C}_A)]^2 \\ &\quad +2[x(\mathrm{C}_{B'}) - x(\mathrm{C}_{A'})](\gamma_v L' - L) - 2v\gamma_v[\tau(\mathrm{C}_B) - \tau(\mathrm{C}_A)] \\ &\quad +L^2 - 2\gamma_v LL' + (L')^2 \end{aligned} \quad (3.6)$$

Setting $\tau(\mathrm{C}_B) = \tau(\mathrm{C}_A)$ and hence, from (2.7), $t'(\mathrm{C}_{B'}) = t'(\mathrm{C}_{A'})$ in (3.6) gives not the relation $L = L'$ but instead the trivial identity:

$$\Delta s = L' = L' \quad (3.7)$$

In fact, the equality $L = L'$ is already implicit in in (3.4) which uses the same spatial coordinate systems in S and S' as (3.2). Since $L = x(\mathrm{C}_{B'}, \tau(\mathrm{C}_B) = 0)$ independently of the value of v, Eqn(3.4) is valid for all values of v. In particular it holds when $v = 0$, $\gamma_v = 1$ and $x \to x'$, in which case it is written:

$$x'(\mathrm{C}_{B'}) - L' = x'(\mathrm{C}_{B'}) - L \quad (3.8)$$

so that

$$L' = L \quad (3.9)$$

A major conceptual error occurs in the interpretation of Figs.2 and 3. It is assumed that these represent observations in the frames S' and S" of the events defined in the frame S in Fig.1. That is, observations of the same events in different frames in the same space-time experiment. If this were indeed the case, then use of the relative velocity transformation formula, Eqn(2.14) of Ref. [24] shows that the speed of μ_{A} and μ_{B} should be $v\gamma_v$ in Fig.2 and $w\gamma_w$ in Fig. 3. It then follows that the claimed 'relativity of simultaneity' effect is annuled and the stated mismatch of spatio-temporal coincidence events (for example, $\mu_{\mathrm{A}'}$ is aligned with B in the frame S, and M in the frame S', when it decays) does not occur. Indeed, as previously correctly pointed out [26, 27], the same spatio-temporal event must occur in all frames from which it may be observed.

What are actually shown in Fig.2 and Fig.3 are the configurations of *physically independent experiments* in these frames. The configuration in Fig. 2 is that of an experiment which is reciprocal to that of Fig. 1 with TD effect given By Eqn(2.8) (clocks in S seen to run slower than clocks in S' in both frames) not that of the primary experiment described by (2.7) in which clocks in S' are seen to run slower than clocks in S in both frames. The configuration of Fig.3 is that of yet another space-time experiment related to that of Fig.1, by a boost with velocity w in the positive x-direction, in which the clocks associated with all the muons are observed to run at the same rate, and to decay simultaneously, in all three frames.

In conclusion, when the experiment is correctly analysed, no 'relativity of simultaneity' or 'length contraction effects occur and the same spatio-temporal coincidence events, at the epochs of the muon decays, are observed in all frames, as they must be.

References

[1] J.H.Field, 'On the Real and Apparent Positions of Moving Objects in Special Relativity: The Rockets-and-String and Pole-and-Barn Paradoxes Revisited and a New Paradox', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0403094v3. Cited 30 Nov 2007.

[2] E.Dewan and M.Beran, Am J. Phys.**27** 517 (1959).

[3] E.Dewan, Am J. Phys. **31** 383 (1963).

[4] V.S.Soni, Eur. J. Phys. **23** 225 (2002).

[5] J.H.Field, 'The Local Space-Time Lorentz Transformation: a New Formulation of Special Relativity Compatible with Translational Invariance', arXiv pre-print: http://xxx.lanl.gov/abs/ physics/0501043v3. Cited 30 Nov 2007.

[6] J.H.Field, 'Uniformly moving clocks in special relativity: Time dilatation, but no relativity of simultaneity or length contraction', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0603135v6. Cited Dec 2008.

[7] J.H.Field, 'Clock rates, clock settings and the physics of the space-time Lorentz transformation', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0606101v4. Cited 4 Dec 2007.

[8] J.H.Field, 'Absolute simultaneity: Special relativity without light signals or synchronised clocks', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0604010v3. Cited 6 Nov 2008.

[9] J.H.Field, Helv. Phys. Acta. **70** 542 (1997), physics/0410262.

[10] V.Berzi and V.Gorini, Journ. Math. Phys. **10** 1518 (1969).

[11] J.H.Field, 'Space-time transformation properties of inter-charge forces and dipole radiation: Breakdown of the classical field concept in relativistic electrodynamics', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0604089v3. Cited 4 Apr 2008.

[12] O.Heaviside, The Electrician, **22** 1477 (1888).

[13] J.H.Field, 'The train/embankment thought experiment, Einstein's second postulate of special relativity and relativity of simultaneity', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0606135v2. Cited 9 Jan 2009.

[14] A.Einstein, 'Relativity, the Special and the General Theory', English translation by R.W.Lawson, Methuen, London, 1960., Ch IX, P25.

[15] H.Poincaré, Rend. del Circ. Mat. di Palermo **21** 129-146, 166-175 (1906).

[16] H.Minkowski, English translation of Address to the 50th Association of German Natural Scientists and Physicians, Cologne, 21st September 1908, in 'The Principle of Relativity' (Dover, New York, 1952) P75.

[17] R.Mansouri and R.U.Sexl, Gen. Rel. Grav. **8**, 497, 515, 809 (1977).

[18] J.H.Field, Phys. Scr. **73** 639 (2006).

[19] J.Bailey *et al* Nature **268** 301 (1979).

[20] R.P.Feynman, R.Leighton and M.Sands, The Feynman Lectures in Physics, Volume I (Addison-Wesley, Reading Massachusetts, 1966) Section 15-3.

[21] E.F.Taylor and J.A.Wheeler, 'Spacetime Physics', W.H.Freeman and Company, San Franciso 1966, Section 42, P89.

[22] P.A.Tipler and R.A.Lewellyn, 'Modern Physics' W.H.Freeman and Company, New York, Ch 1, P40.

[23] A.Walker, CERN Courier, **46** Number 4 May 2006, P23.

[24] J.H.Field, 'Muon decays in the Earth's atmosphere, differential aging and the paradox of the twins', arXiv pre-print: http://xxx.lanl.gov/abs/0809.1314. Cited 8 Sep 2008.

[25] J.H.Field,'Muon decays in the Earth's atmosphere, time dilatation and relativity of simultaneity', arXiv pre-print: http://xxx.lanl.gov/abs/physics/0606188v2. Cited 30 Oct 2006.

[26] P.Langevin, Scientia **10** 31 (1911) P41.

[27] N.D.Mermin, Am. J. Phys **51** 1130 (1983), **52** 119 (1984)